Friction Damping of Flutter in Gas Turbine Engine Airfoils

A. Sinha* and J. H. Griffin†
Carnegie-Mellon University, Pittsburgh, Pennsylvania

This paper investigates the feasibility of using blade-to-ground friction dampers to stabilize flutter in blades. The response of an equivalent one mode model in which the aerodynamic force is represented as negative viscous damping is examined to investigate the following issues: the range of amplitudes over which friction damping can stabilize the response, the maximum negative aerodynamic damping that can be stabilized in such a manner, the effect of simultaneous resonant excitation on these stability limits, and the determination of those damper parameters which will be the best for flutter control.

I. Introduction

AEROELASTIC instability or flutter is a self-excited oscillation that occurs when the unsteady work, W_{FB}, performed by the fluid on the blade exceeds the energy dissipated by damping in the system. This is an important problem in the development of modern gas turbine engines. Manufacturers expend considerable effort in devising flutter prediction systems which are then used to design airfoils.[1] In theory, the cascade so designed will have positive aerodynamic damping, $W_{FB}<0$, over the entire compressor operating map. In spite of such precautions, flutter occurs in new engines even when no such problem is anticipated on the basis of analytical predictions and rig testing.[2] One solution which is being pursued is to evolve better flutter prediction codes based on more comprehensive analyses.[3,4] Another solution proposed in this paper is to increase the amount of mechanical damping in the system by the use of friction damping. A friction damper provides a link between points experiencing relative motion caused by vibration. It transmits a load through friction contacts which dissipate energy when slip occurs. While dampers have been used for years by manufacturers to control resonant vibratory stresses in turbines, the feasibility of using them to improve compressor aeroelastic stability has not been previously reported. The objective of this paper is to investigate under what circumstances such devices might prove useful and what would be the primary considerations in their design.

Specifically, this paper is concerned with the dynamic response of blades having blade-to-ground friction dampers, Fig. 1. This type of damper provides a link between a vibrating point on the blade and a relatively rigid structure, for example, the platform and the cover plate. Such a damper was studied with regard to its effectiveness in reducing turbine resonant stresses.[5] In the typical case of turbine resonance where it is necessary to use dampers, the aerodynamic forces provide positive, but inadequate, damping. Under these conditions, the objectives of a damper analysis are to calculate the amplitude and phase of the frictionally damped blade and to determine what values of damping parameters will result in the minimum amplitude of response. Such was the subject of Ref. 5. In contrast, flutter, the topic of this paper, is a self-excited oscillation usually encountered in fans or compressors. In the case of flutter the objectives of a damper analysis are to determine if it can be controlled by the damper and, if it can, what is the amplitude and frequency of the response and under what conditions would it become unstable? In Ref. 5 it was found that it was important to model the flexibility of the damper since the blade's displacements were so small that frequently the damper was not slipping but merely deflecting. The inertia of the damper on the other hand was usually negligible since the important lowest mode responses were at relatively low frequencies. These assumptions will be carried over to this study and the damper will be modeled as the massless spring of Fig. 1.

First, the equivalent single mode model for the blade-to-ground damper is described and its limitations discussed. Its approximate steady-state response is calculated using the Ritz procedure.[6] For first-order steady response the Ritz method is equivalent to the alternate methods described in Refs. 7-9. The validity of the approximate solution is corroborated by numerical integration of the governing differential equation. In this paper, on the basis of this single-degree-of-freedom model, the following four basic issues have been addressed for an unstable system (linear damping<0): 1) the range of initial conditions over which the response is stable, 2) the maximum negative aerodynamic damping that can be stabilized by friction, 3) the effect of simultaneous resonant excitation on the stability of the system, and 4) for a fixed excitation level, the determination of an optimum slip load to minimize the vibratory stress.

II. Analysis and Discussion of Results

A. Single-Degree-of-Freedom Model

An equivalent nonlinear single-degree-of-freedom system, as shown in Fig. 2, was derived[5] by representing the blade's motion in terms of its fundamental modes. This model was previously used to predict the slip load at which a blade's resonant amplitude would be minimized. The analytically predicted results compared well with experimental data, particularly when the damper stiffness was moderate. For stiffer dampers the blade's mode shape can change significantly as the damper slip force changes. Obviously, in this case, a multiple-degree-of-freedom model, similar to those developed by Muszynska,[10] that also can change mode shape would more accurately simulate the blade's response over the entire range of possible slip loads. However, even for stiff dampers the single mode model can provide lower and upper bounds on the blade's response by performing two analyses based on the blade's mode shapes corresponding to the fully slipping and stuck dampers, respectively. In this sense, the single mode model represents, at least qualitatively, the full range of possible damper stiffnesses. The advantage of analyzing the simpler representation is that the relatively simple physics of the system permits a general physical interpretation of the results and at least a first-order approximation to a general solution of this generic class of dynamics problems.

Nondimensionalization of nonlinear problems makes it easier to numerically verify analytical results and allows a more concise presentation of data. The dimensional system is depicted in Fig. 2, F_d is the force required to cause the damper

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*Research Assistant. Department of Mechanical Engineering.

†Assistant Professor. Department of Mechanical Engineering.

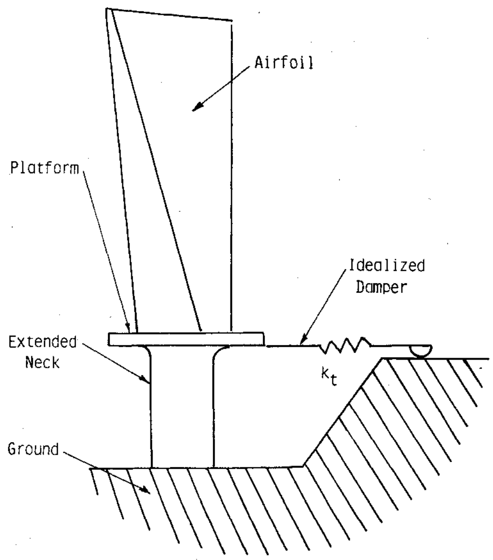


Fig. 1 Blade to ground damper.

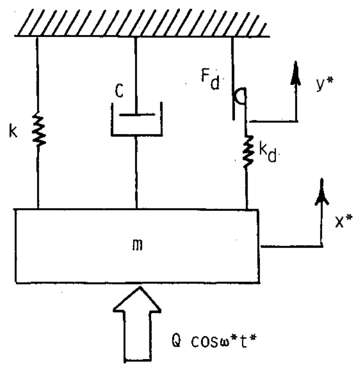


Fig. 2 Equivalent 1-degree-of-freedom system.

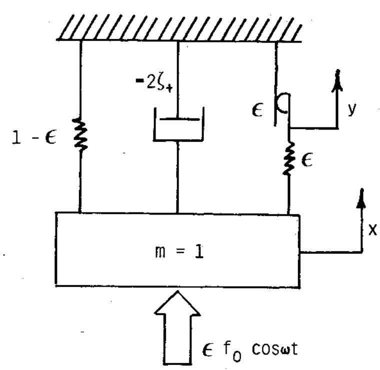


Fig. 3 Dimensionless system.

to slip. Introducing the following notation:

$$\epsilon = k_d/(k_d+k) \qquad \zeta_+ = -c/[4m(k+k_d)]^{1/2}$$

$$x = x^*/x_0 \qquad y = y^*/x_0$$

where $x_0 = F_d/k_d$;

$$t = t^*/T \qquad \omega = \omega^* T \tag{1}$$

where $T = [m/(k+k_d)]^{1/2}$;

$$f_0 = Q/F_d = Q/(k_d x_0)$$

the differential equation of motion is

$$\frac{d^2x}{dt^2} - 2\zeta_+ \frac{dx}{dt} + x = \epsilon y + \epsilon f_0 \cos\omega t \tag{2}$$

The system corresponding to Eq. (2) is shown in Fig. 3. Slip only can occur when $|x-y| = 1$. ϵ is the fraction of the system's stiffness attributable to the damper and ζ_+ is the amount of negative aerodynamic damping in the system as a fraction of critical damping ($\zeta_+ > 0$ corresponds to aeroelastic instability, $W_{FB} > 0$). If the slip load is fixed, then f_0 is directly proportional to the excitation level. Alternately, it is inversely proportional to slip load for a fixed excitation level. In this case the actual displacement is proportional to x/f_0.

If the slip load is infinite, the system would be always stuck, i.e., $y = 0$. On the other hand, $y = x$ if the slip load is zero. The system is linear for both these extreme values of slip load and would be unstable for $\zeta_+ > 0$. In certain instances, the unstable linear system can be stabilized by a choice of slip load between its extreme values of zero and infinity since there will be a dissipation of energy because of slip at the friction joint. The presence of slip makes the system nonlinear and the stability analysis more complex. The stability of the system depends on the initial conditions specified as well as the magnitude and frequency of any external excitation, if present. The system is likely to be unstable for large initial displacements because the work done by aerodynamic forces (which is proportional to the square of amplitude) would become greater than that dissipated by the damper (which is proportional to amplitude). However, even if the initial displacements are small, the system may exhibit unbounded response if the magnitude of the external excitation is large compared to the slip load. This phenomenon of unbounded resonance has been described by Den Hartog[11] and it occurs when the work done by the excitation force is larger than the energy dissipated by friction with a small slip load. In the next section the simple initial value problem, $f_0 = 0$, will be considered. The impact of external excitation on stability will be examined subsequently.

B. Initial Value Problem ($f_0 = 0$)

If an initially quiescent system is perturbed, its response will grow exponentially until slip occurs. The response is likely to attain a steady state when the work done by aerodynamic forces equals the energy dissipated by friction. Assume that the steady-state response is approximately harmonic,

$$x = A\cos\omega t \tag{3}$$

Here ω is the natural frequency of the system and is not known a priori for this nonlinear system. Under the assumption that the response is harmonic, the nonlinear term ϵy can be expanded in a Fourier series, the first terms of which are (see the Appendix)

$$\epsilon y = a\cos\omega t + b\sin\omega t \tag{4}$$

where

$$a = \epsilon A[\pi - \theta_c + \tfrac{1}{2}\sin 2\theta_c]/\pi \tag{5}$$

$$b = 4\epsilon(1 - 1/A)/\pi \tag{6}$$

and

$$\theta_c = \cos^{-1}(1 - 2/A) \tag{7}$$

By substituting Eqs. (3) and (4) into Eq. (2), it can be seen that the cosine component of y adds to the stiffness of the system and the sine component to the aerodynamic damping term. Hence the system under consideration has both nonlinear damping and a nonlinear restoring force. As a matter of fact, $b/(\omega A)$ turns out to be the equivalent viscous damping due to slip at the friction joint in the sense of Den

Hartog.[11] Since for steady-state response, the aerodynamic and frictional forces balance

$$2\zeta_+ = b/(\omega A) \tag{8}$$

From Eqs. (6) and (8),

$$A = \frac{4/\pi \pm [(4/\pi)^2 - (32\zeta_+\omega)/(\epsilon\pi)]^{1/2}}{4(\zeta_+\omega/\epsilon)} \tag{9}$$

From Eqs. (2), (4), and (5),

$$\omega^2 = 1 - \epsilon(\pi - \theta_c + 1/2 \sin 2\theta_c)/\pi \tag{10}$$

Equations (9) and (10) can be solved simultaneously for a given value of ϵ and different values of ζ_+. Typically the contribution of the damper to the system's stiffness is relatively small and consequently ϵ is a small fraction. For small ϵ, the flutter frequency is to first-order unity and A is a function only of ζ_+/ϵ. The amplitude, A, vs ζ_+/ϵ is depicted in Fig. 4 for ϵ small. For each value of ζ_+/ϵ, there are two values of A at which the aerodynamic work equals the energy dissipated. The lower value, A_L, turns out to be the stable steady-state solution as long as the initial displacement is lower than the upper value of A, A_u. The system is unstable when the initial displacement is larger than A_u and the initial velocity is zero. This behavior is illustrated in Figs. 5 and 6 which are responses obtained from numerical integration for initial conditions corresponding to points 1 and 2 in Fig. 4. It is to be noted that there is a critical value of ζ_+ beyond which the system is always unstable.

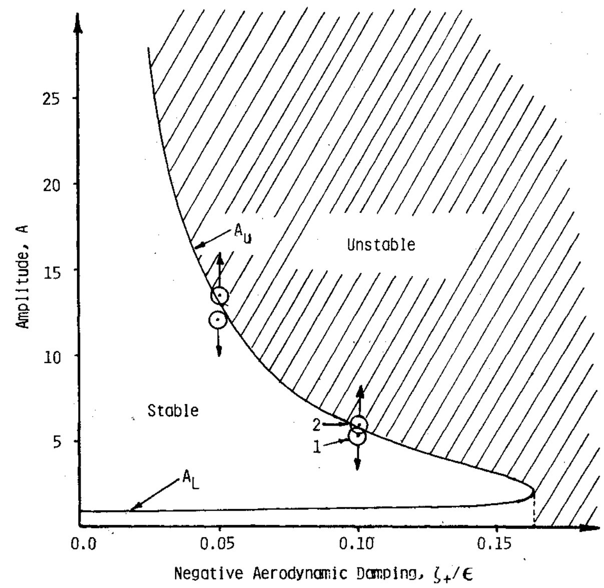


Fig. 4 Stability plot for no external excitation.

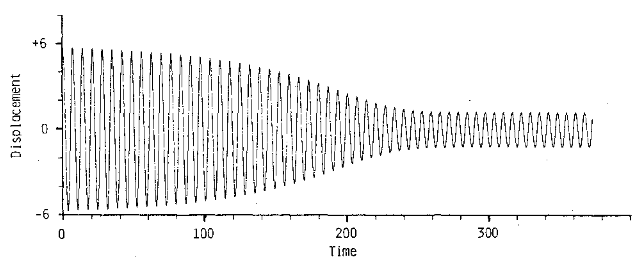


Fig. 5 Point 1 response, stable initial conditions.

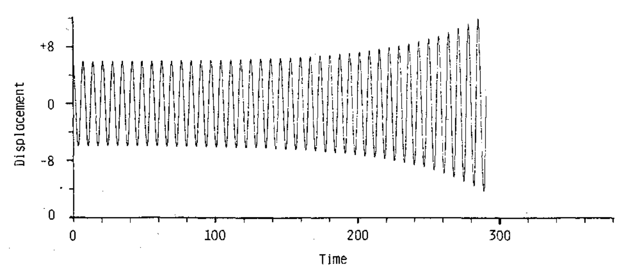


Fig. 6 Point 2 response, initial conditions not stable.

Also, it can be seen that the point $x = \dot{x} = 0$ in the phase plane is a singular point for the governing differential equation, Eq. (2) ($y = 0$). For negative values of aerodynamic damping, the singular point is found to be an unstable focus[12] as shown in Fig. 7. The lower value of A in Fig. 4 corresponds to the limit cycle in the phase plane towards which the neighboring trajectories spiral on both sides. The stability boundary has been obtained from the relation

$$k_{eq}x^2(0) + \dot{x}^2(0) = k_{eq}A_u^2 \tag{11}$$

where $1 - \epsilon \leq k_{eq} \leq 1$, which was derived from energy considerations. A conservative estimate of the stability boundary is obtained by substituting $k_{eq} = 1 - \epsilon$ in Eq. (11). Numerical time integration was used to independently confirm that Eq. (11) does define the stability boundary.

The two damper characteristics, slip load and stiffness, play interesting roles in terms of damper effectiveness and system stability. It is apparent from Eq. (1) that the actual displacement is proportional to slip load times A. Thus selecting a slip load is equivalent to scaling the vertical axis in Fig. 4. For fixed ζ_+/ϵ selecting a larger value of slip load proportionately increases the physical displacements corresponding to A_u and A_L. From a design point of view, increasing the stability limit is beneficial since the blade will be able to sustain larger transient motions without going unstable. However, increasing the physical displacement corresponding to A_L is generally undesirable since it will yield larger steady-state amplitudes. As a result, selection of an optimum slip load is a tradeoff that must be made by the designer with the vibratory and fatigue characteristics of the blade in mind.

Also of interest is the fact that the damper's stiffness parameter, ϵ, scales the horizontal axis in Fig. 4. As a result, the stiffer the damper the greater the amount of negative aerodynamic damping it can control. The damper's modal stiffness, k_d of Fig. 2, is equal to its physical stiffness multiplied by the square of the blade's normalized modal displacement at the damper contact point. From a design point of view, ϵ $(= k_d/(k + k_d))$ and the damper's ef-

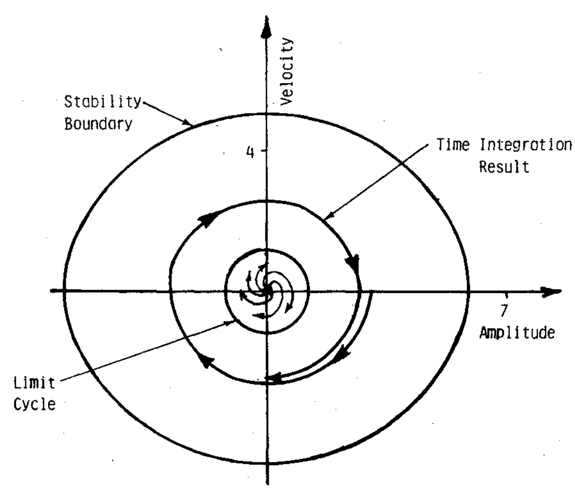


Fig. 7 Phase plane plot.

fectiveness can be increased by either increasing its physical stiffness or increasing the blade's relative displacement at the point of damper contact. For example, for a platform-to-cover plate damper the platform to blade tip displacement ratio can usually be increased by lengthening the extended neck (Fig. 1).

The maximum value of ζ_+ that can be controlled by a friction damper, $\zeta_{+(max)}$, can be inferred from Eq. (9). For this value of ζ_+, the amplitude is single valued and the quantity under the radical sign must be zero. For ϵ small

$$\zeta_{+(max)} \cong \epsilon/2\pi \tag{12}$$

If it is assumed that compressor dampers will exhibit the same range of ϵ as those in turbines, then experience would indicate that a rough estimate of $\zeta_{+(max)}$ as a fraction of critical damping is

$$0.01 < \zeta_{+(max)} < 0.04 \tag{13}$$

depending on the particular geometry of the blade-damper system. Such amounts of friction induced damping is significant and in many instances should prove a useful deterrent to flutter.

C. Forced Response with Negative Aerodynamic Damping

In practice, there is always some forcing function due to circumferential perturbations in the pressure field caused by the presence of upstream static structures. When the aerodynamic damping is negative, the forced response will attain a steady state when the energy dissipated by friction equals the work done by aerodynamic and external forces over a period of response. In general, it is not straightforward to calculate the response at an arbitrary excitation frequency because the steady state motion of the blade, while periodic, is not simply harmonic. It is composed of two components, a flutter induced response at the natural frequency of the system and a forced response at the excitation frequency. Fortunately, the most significant case corresponds to an external excitation at the resonant frequency, a situation in which the response is simply harmonic. In this instance the steady-state amplitude can be calculated in the same way as was done for positive aerodynamic damping.[5] Specifically, if it is assumed that the steady-state response is

$$x = A\cos(\omega t + \phi) \tag{14}$$

then y may be expanded in the same manner as before, the first terms of which are

$$\epsilon y = a\cos(\omega t + \phi) + b\sin(\omega t + \phi) \tag{15}$$

where, since y is only a function of x, a and b still are given by Eqs. (5) and (6). If Eqs. (14) and (15) are substituted into the governing differential equation (2), a harmonic balance implies that at resonance ϕ equals $-\pi/2$ and

$$A = \frac{(4/\pi - f_0) \pm [(4/\pi - f_0)^2 - 32\zeta_+\omega/\epsilon\pi]^{1/2}}{4\zeta_+\omega/\epsilon} \tag{16}$$

and ω is given by Eqs. (10) and (7). For bounded response,

$$f_0 \leq 4/\pi - (32\zeta_+\omega/\epsilon\pi)^{1/2} \tag{17}$$

Equations (10) and (16) are solved simultaneously for A and ω. Results for ϵ small are shown in Fig. 8 as A vs ζ_+/ϵ

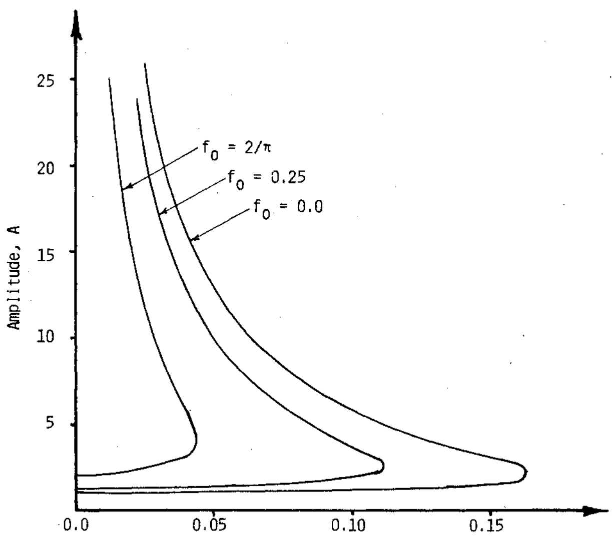


Fig. 8 Effect of external excitation in stability plot.

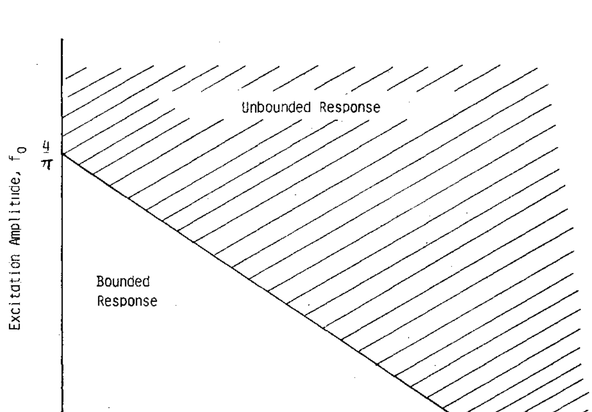


Fig. 9 Maximum excitation for stable response.

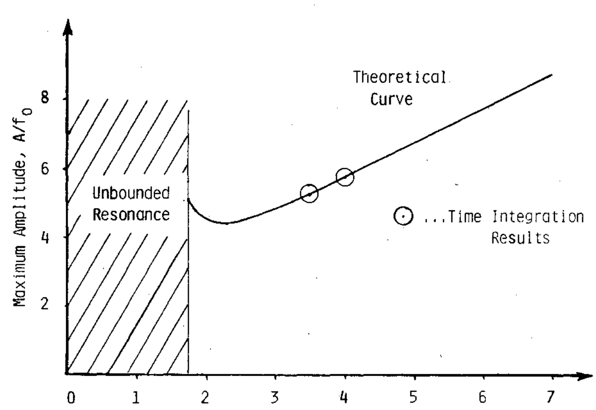
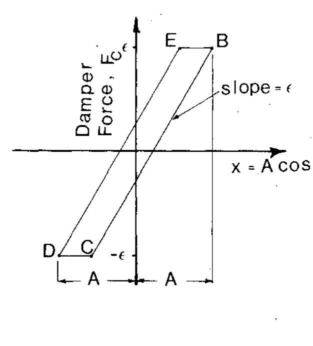


Fig. 10 Maximum response vs slip load.

Fig. 11 Damper force vs displacement.



plots for various values of f_0. Again, numerical time integration corroborates that the lower values of A correspond to stable steady-state response. The upper values of A provide a conservative estimate of the stability boundary. As indicated in Fig. 9 for small ϵ, i.e., $\omega \cong 1$, Eq. (17) gives the maximum value of f_0 as a function of ζ_+ / ϵ for which the response will be bounded. It is interesting to note that if $f_0 > 4/\pi$ the response will always be unbounded for $\zeta_+ \geq 0$. This critical condition $f_0 = 4/\pi$ is the same as that obtained by Caughey[7] for $\zeta_+ = 0$.

D. Optimization of Slip Load for a Fixed Excitation Level

For a given excitation level, there exists an optimum value of slip load which will minimize the resonant response. For an aerodynamically unstable system, the resonant amplitude, A, is given by Eq. (16) with a minus sign in front of the radical. Responses were obtained numerically at frequencies other than the natural frequency of the system and, as expected, the resonant amplitude so calculated does correspond to the maximum response for a given f_0. For a fixed excitation level, the maximum vibratory stress is proportional to A/f_0 and the slip load is proportional to $1/f_0$. A plot of A/f_0 vs $1/f_0$ is shown in Fig. 10 for $\epsilon = 0.2$ and $\zeta_+ = 0.01$. It can be seen that the optimum slip load is close to a region of unbounded response. Hence, from a design point of view, it would be reasonable to select a slip load larger than the optimum value to allow for some margin of error.

III. Conclusions

Values of initial velocity and amplitude for which the response would be stable were determined as a function of a negative aerodynamic damping parameter, ζ_+. There is a maximum value of ζ_+, $\zeta_{+(\max)}$, for which the system can be stabilized by friction. It is sufficiently large that the use of friction dampers for flutter control appears promising and should receive further study.

For ζ_+ less than $\zeta_{+(\max)}$, there are two values of amplitude at which the aerodynamic work is equal to the energy dissipated by friction (Fig. 4). The smaller amplitude corresponds to the steady-state response, while the larger is a stability limit. Both of these amplitudes are proportional to the damper's slip load. Consequently from a design point of view, selecting the damper's slip load establishes the steady-state amplitude of the blade (which is important from a high cycle fatigue point of view) and the maximum transient response for which the blade will be stable. A second observation is that $\zeta_{+(\max)}$ is proportional to the fraction of the system's stiffness due to the damper. And, as a result, increasing the damper's stiffness increases its effectiveness.

For a fixed value of ζ_+, the addition of resonant excitation decreases the maximum amplitude at which the system will be stable as indicated in Fig. 8. If the amplitude of the excitation is greater than $4/\pi$ times the slip load, then the response is unbounded for all $\zeta_+ \geq 0$. For a fixed level of excitation and $\zeta_+ < \zeta_{+(\max)}$, the damper slip load can be chosen to minimize the amplitude of response. Since the optimum slip load is near a condition of unbounded response it would be reasonable to provide a margin of safety by selecting a slip load greater than the mathematical optimum.

Appendix: Evaluation of Nonlinear Term, y

From Fig. 3, the compressive force in the damper spring, F_c, varies as a function of x as indicated in Fig. 11 when the amplitude of x is greater than 1. If x equals $A\cos\theta$, where θ is $\omega t + \phi$, y can be expanded in a Fourier series having the same argument, θ, as x. First $y(\theta)$ is expressed as a function of x by using the damper force vs displacement relationship depicted in Fig. 11 and also the fact that F_c equals $\epsilon(x-y)$. After some simple algebra, on BC:

$$y = A - 1 \qquad 0 \leq \theta \leq \theta_c$$

and on CD:

$$y = x + 1 \qquad \theta_c \leq \theta \leq \pi \tag{A1}$$

where $\theta_c = \cos^{-1}(1 - 2/A)$ is the value of θ corresponding to point C in Fig. 11.

It is found that $y(\theta) = -y(\theta + \pi)$; and the period of y is the same as that of x. Representing ϵy as a Fourier series and keeping only the fundamental component,

$$\epsilon y = a\cos\theta + b\sin\theta \tag{A2}$$

where

$$a = (2\epsilon/\pi) \int_0^\pi y\cos\theta \mathrm{d}\theta \tag{A3}$$

and

$$b = (2\epsilon/\pi) \int_0^\pi y\sin\theta \mathrm{d}\theta \tag{A4}$$

The integrals in Eqs. (A3) and (A4) are evaluated using Eq. (A1) and standard tables. The results are given in Eqs. (5) and (6). Since y is only a function of x and the trigonometric functions in Eq. (A2) have the same argument, θ, as x, these results hold for all values of the phase angle ϕ.

Acknowledgments

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